Thermodynamics with Highly Improved staggered quark (HISQ) action

Péter Petreczky and Alexei Bazavov
(HotQCD Collaboration)

BROOKHAVEN NATIONAL LABORATORY

Goal : 1st principle LQCD study of the phase diagram and properties of hot strongly interacting matter at not too high baryon density with controlled discretization errors

Continuum limit : $a \to 0, \quad N_\tau \to \infty$ $\qquad a = 1/(TN_\tau)$

Computational costs: $\sim a^{-8}, \quad \sim N_\tau^8$ ⇒ use the least demanding staggered fermion formulation and large computers: BG/L (NYBlue), USQCD clusters

low T region (T<200 MeV) $\mathcal{O}(\alpha_s^n (a\Lambda_{QCD})^2)$ errors

high T region (T>200MeV) $\mathcal{O}((aT)^2)$ errors

need to reduce both type of errors => HISQ action

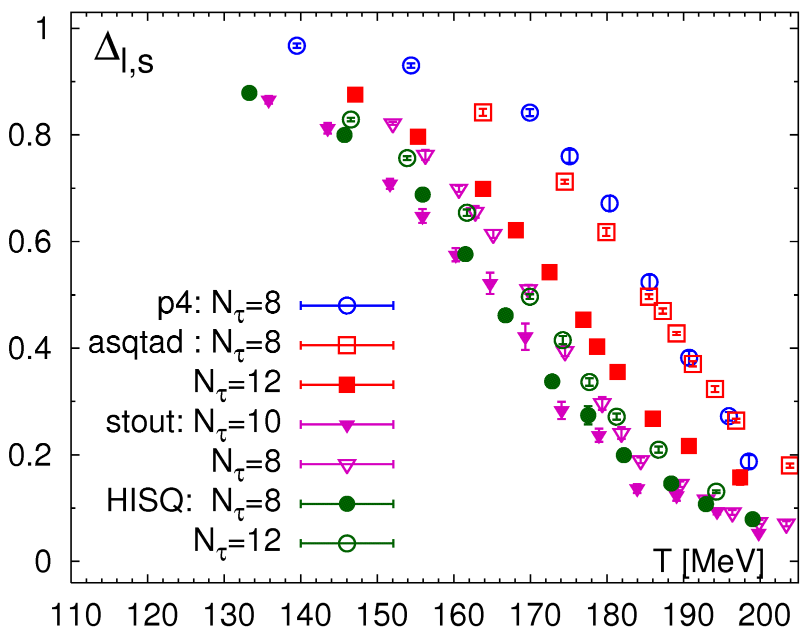
DNP 2010, Santa Fe, November 3-6, 2010

Chiral and deconfinement transition in QCD

The QCD transition at *T>0* is a crossover !

Chiral transition : drop in chiral condensate

$$\Delta_{s,l}(T) = \frac{\langle\bar\psi\psi\rangle_T - \frac{m_l}{m_s}\langle\bar s s\rangle_T}{\langle\bar\psi\psi\rangle_{T=0} - \frac{m_l}{m_s}\langle\bar s s\rangle_{T=0}}$$



Bazavov, P.P., arXiv:1009.4914; arXiv:1005.1131

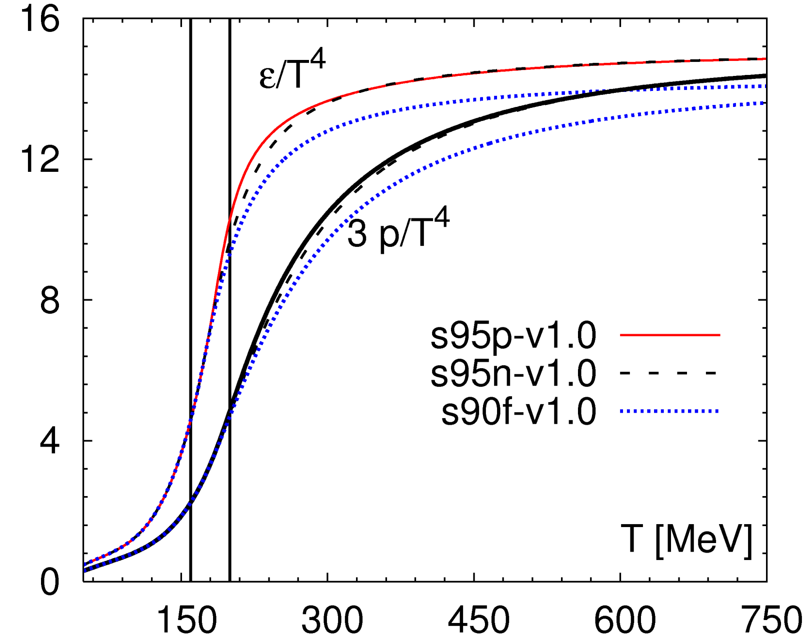
Chiral transition temperature:

T_c=(147-157) MeV (Budapest-Wuppertal)

T_c=164(6) MeV (HotQCD preliminary, Lat'10)

Deconfinement : rapid increase in the energy density for $T \approx$ *(160-200)* MeV

Lattice+HRG, Huovinen, P.P., NPA 837 (10) 26



Pseudo-critical energy density :

$$\epsilon(T_c) \simeq 300 \mathrm{MeV}/fm^3$$

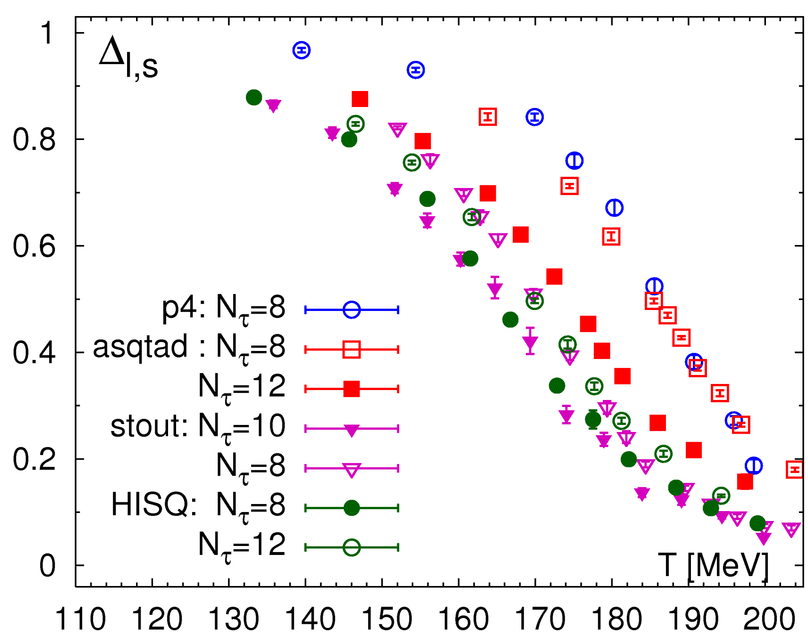
too low for existence of partonic dof

Chiral and deconfinement transition in QCD

The QCD transition at *T>0* is a crossover !

Chiral transition : drop in chiral condensate

$$\Delta_{s,l}(T) = \frac{\langle\bar\psi\psi\rangle_T - \frac{m_l}{m_s}\langle\bar s s\rangle_T}{\langle\bar\psi\psi\rangle_{T=0} - \frac{m_l}{m_s}\langle\bar s s\rangle_{T=0}}$$



Bazavov, P.P., arXiv:1009.4914; arXiv:1005.1131

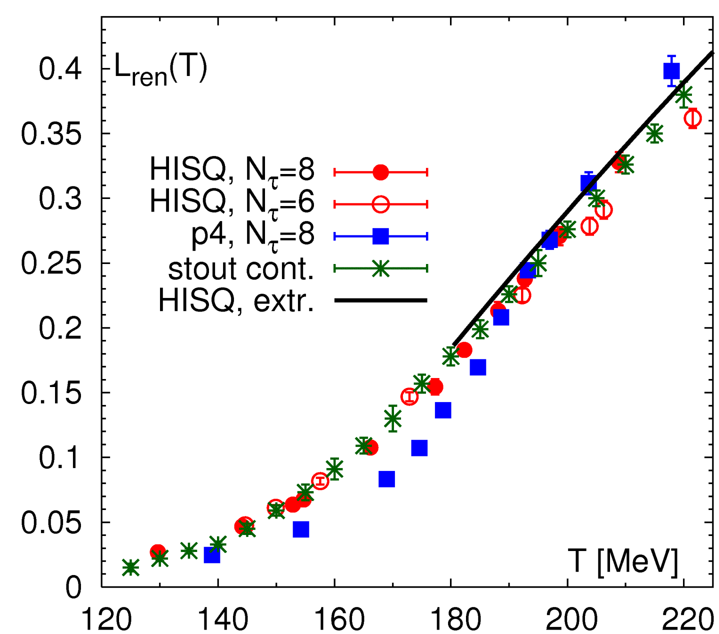
Chiral transition temperature:

T_c=(147-157) MeV (Budapest-Wuppertal)

T_c=164(6) MeV (HotQCD preliminary, Lat'10)

Deconfinement : increase in the static free energy of a static quark

Polyakov loop $L_{ren} = \exp(-F_Q(T)/T)$



low T :

$F_Q(T) \sim 2M_{D,B} - m_{c,b} \simeq \Lambda_{QCD}$

high T :

$F_Q(T) \sim -\alpha_s m_D$ screening !

Lattice results on the trace of energy momentum tensor

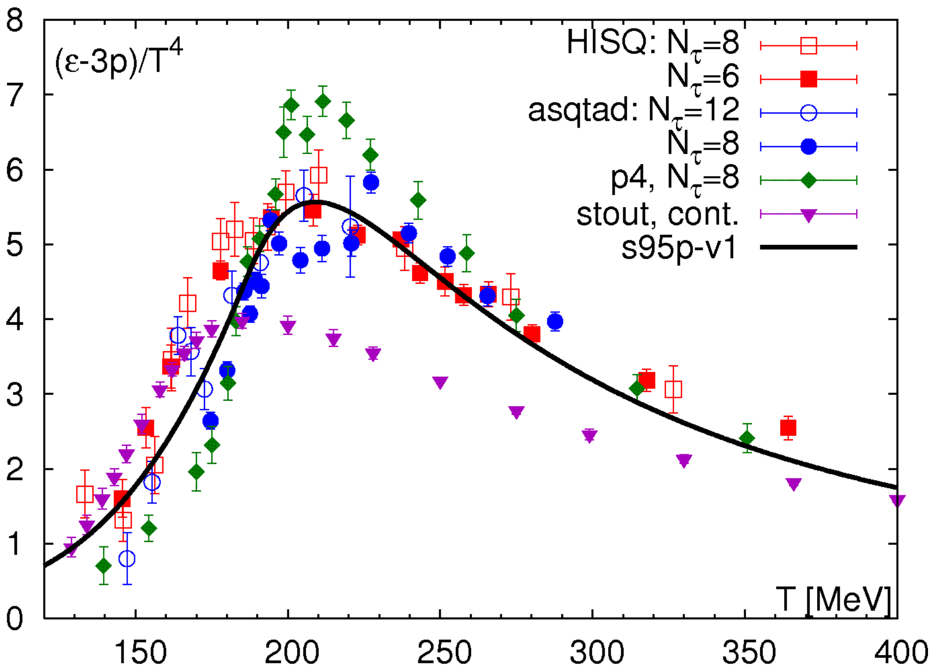
EoS is calculated from the trace anomaly $\Theta^{\mu\mu}(T) = \epsilon - 3p$ (integral method)

$$\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \frac{\Theta^{\mu\mu}(T')}{T'^5},$$

Bazavov, P.P., arXiv:1005.1131
Bazavov, P.P., Lattice 2010

For weakly interacting quarks and gluons:

$$\epsilon - 3p \sim \alpha_s^2(T)T^4 \ll T^4$$



- HISQ results on the trace anomaly agree with previous HotQCD results for *T>250*MeV
- A better agreement is achieved with HRG in the low *T* region with the HISQ action
- The HISQ results are compatible with asqtad calculations in the peak region
- HISQ results agree quite well with s95p-v1 parametrization of EoS that is based on HRG+LQCD and used in hydro models

Huovinen, P.P., NPA 837 (10) 26

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_S,\mu_I)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!l!} \chi_{ijk}^{BSI} \cdot \mu_B^i \cdot \mu_S^j \cdot \mu_I^k$$ hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{uds} \cdot \mu_u^i \cdot \mu_d^j \cdot \mu_s^k$$ quark

$$\chi_{ijk}^{abc} = \frac{\partial^i}{\partial \mu_a^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{1}{VT^3} \ln Z(T,V,\mu_a,\mu_b,\mu_c)|_{\mu_a=\mu_b=\mu_c=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

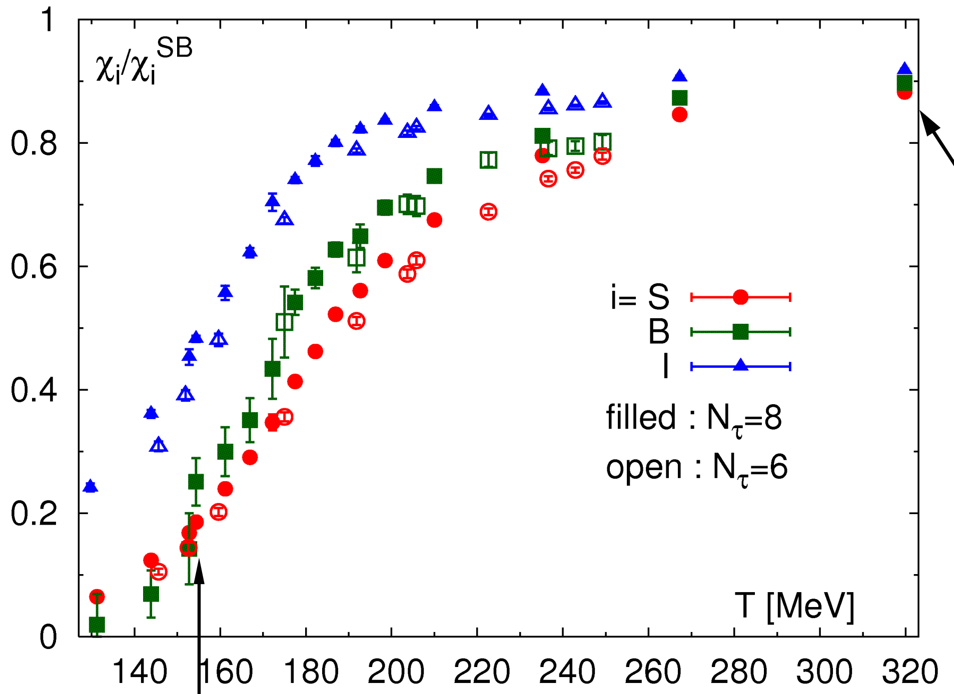
$$\frac{\chi_2^X}{T^2} = \frac{\chi_X}{T^2} = \frac{1}{VT^3}(\langle X^2\rangle - \langle X\rangle^2) \qquad \frac{\chi_{11}^{XY}}{T^2} = \frac{1}{VT^3}(\langle XY\rangle - \langle X\rangle\langle Y\rangle)$$

Deconfinement : fluctuations of conserved charges

$$\frac{\chi_B^{SB}}{T^2} = \frac{1}{VT^3}(\langle B^2 \rangle - \langle B \rangle^2)$$ baryon number

$$\frac{\chi_I^{SB}}{T^2} = \frac{1}{VT^3}(\langle I^2 \rangle - \langle I \rangle^2)$$ isospin

$$\frac{\chi_S^{SB}}{T^2} = \frac{1}{VT^3}(\langle S^2 \rangle - \langle S \rangle^2)$$ strange quark number (strangeness)



conserved charges are carried by massive hadrons

Ideal gas of quarks :

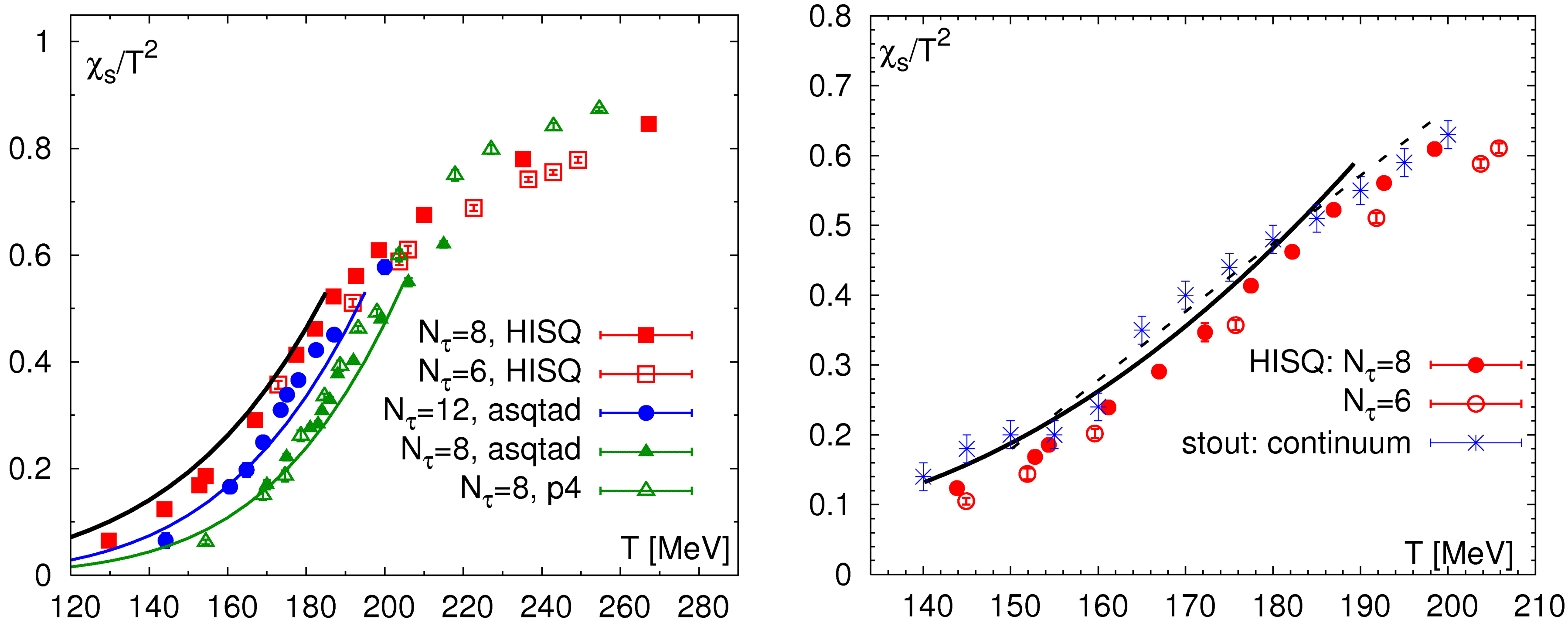
$$\chi_B^{SB} = \frac{T^2}{3} \quad \chi_I^{SB} = \frac{T^2}{2}$$

$$\chi_B^{SB} = T^2$$

conserved charges carried by light quarks

Bazavov, P.P., arXiv:1005.1131

Strangeness fluctuations and HRG



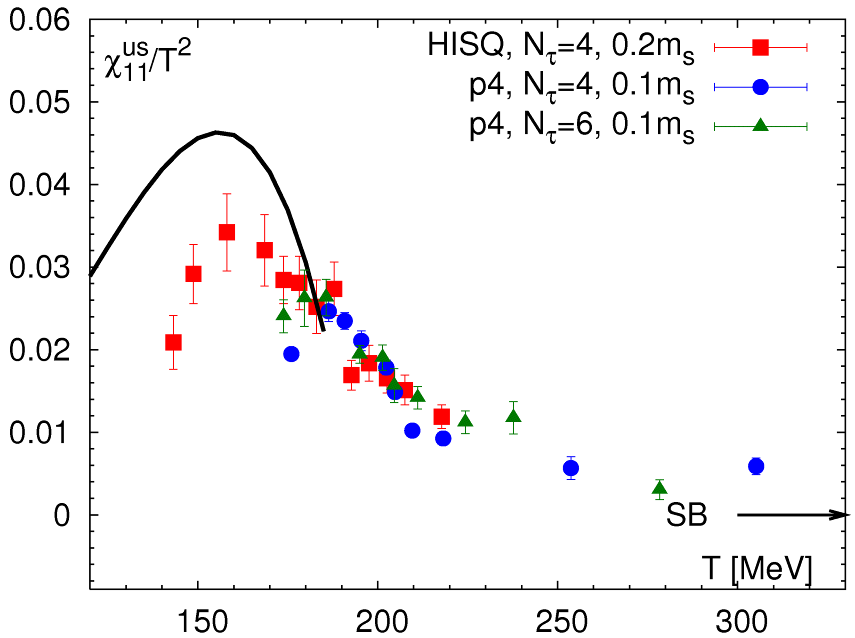
enhancement of strangeness fluctuations compared to the previous calculations on *$N_\tau=8$* with lattice due to reduced discretization errors

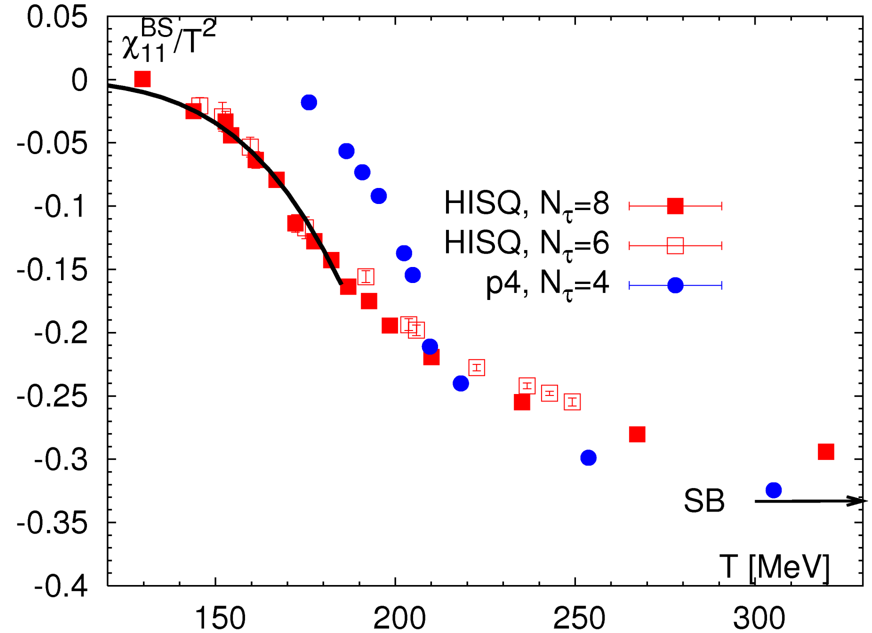
All the lattice results are below the HRG. Hadron masses on the lattice are large than physical due to a^2 discretization effects this needs to be taken into account =>
good agreement between lattice and modified HRG, Huovinen, P.P, arXiv:0912.2541
Langelage, Philipsen arXiv:1002.1507

lattice calculations with stout and HISQ action are expected to give consistent results in the continuum limit and agree with physical HRG

stout continuum : arXiv:1005.3508

Correlations of conserved charges





Bazavov, P.P., arXiv:1005.1131

- Correlations between strange and light quarks at low *T* are due to the fact that strange hadrons contain both strange and light quarks but very small at high *T (>250* MeV*)*
=> weakly interacting quark gas

- For baryon-strangeness correlations HISQ results are close to the physical HRG result, at *T>250* MeV these correlations are very close to the ideal gas value

- The transition region where degrees of freedom change from hadronic to quark-like is broad ~ 50 MeV

Summary

• Lattice calculations with HISQ action largely reduce cutoff effects in thermodynamic quantities and make possible to reach the continuum limit with currently available computational resources.

• Calculations of several finite temperature observables (Polyakov loop, chiral condensate and strangeness fluctuation) obtain with different staggered fermion actions seem to agree as we approach the continuum limit

• The chiral transition takes place at temperature *(150 -165)* MeV

• At low *T* thermodynamics can be understood in terms of hadron resonance gas The deconfinement transition can understood as transition from hadron resonance gas to quark gluon gas it is gradual and analagous to ionized gas – plasma transition, and happens at temperatures higher than the chiral transition temperature as first suggested by Wuppertal-Budapest Collaboration

• We need to work on lattices $N_\tau \geq 8$ (maybe $N_\tau = 6$ for HISQ) to get close to the continuum limit

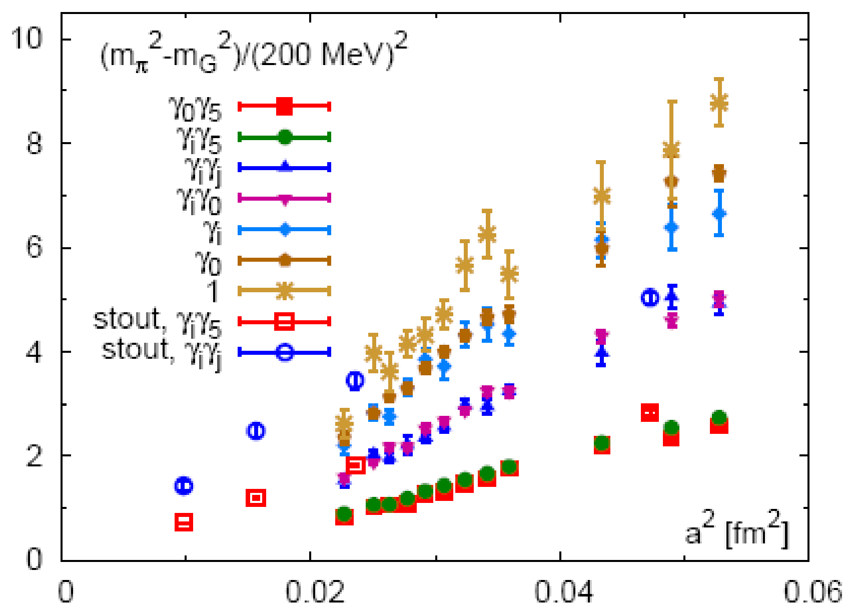
Back-up: Calculations with HISQ action

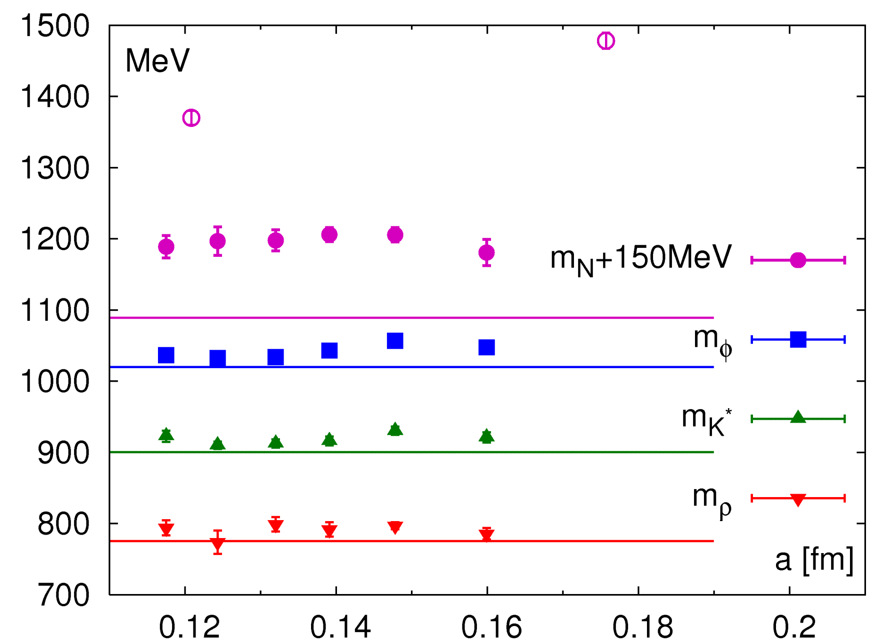
m_s physical (at 1%), $m_l = 0.05 m_s$, 32^4, 32^3x8 and 24^3x6 lattices, $L_s = (3.2\text{-}5.0) fm$

The lattice spacing is set through the calculation of static potential

$$\left(\frac{dV_{q\bar{q}}(r)}{dr} \right)_{r=r_0} = 1.65, \quad r_0 = 0.469(7) fm$$

$$m_\pi = 159(1) \text{ MeV } (140 \text{ MeV})$$
$$m_K = 499(3) \text{ MeV } (494 \text{ MeV})$$
$$m_{\eta_{ss}} = 689(3) \text{ MeV } (686 \text{ MeV})$$





- The potential shows little cutoff dependence and similar to that calculated with asqtad and p4
- Significant improvement in hadron spectrum compared to asqtad calculations => smaller cutoff effects at small *T*, independent consistency check on the scale setting